Guided Notes — Finding Roots

the factoring method

Example 1

Find the square root of 300.

$\boxed{\text{step 1}}$: Break down 300 using prime factors

→ option 1 : upside down division

→ option 2 : factor tree

$$\begin{array}{r|l} 2 & 300 \\ 2 & 150 \\ 3 & 75 \\ 3 & 25 \\ & 5 \end{array}$$

so $300 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$

$\boxed{\text{step 2}}$: Findany prefect square factors.
(in this case ($\sqrt{\ }$) → groups of 2)

$$300 = (2 \cdot 2) \cdot (3 \cdot 3) \, 5 = \underbrace{4 \cdot 9}_{\text{perfect squares}} \cdot 5$$

$\boxed{\text{step 3}}$: simplify

$$\sqrt{300} = \sqrt[2]{(2 \cdot 2) \cdot (3 \cdot 3) \cdot 5} = 2 \cdot 3\sqrt{5} = \boxed{6\sqrt{5}}$$

Example 2 Find the square root of 320

$$\begin{array}{r|l} 2 & 320 \\ 2 & 160 \\ 2 & 80 \\ 2 & 40 \\ 2 & 20 \\ 2 & 10 \\ & 5 \end{array}$$

$$\sqrt{320} = \sqrt{2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot5} = 2\cdot2\cdot2\sqrt{5} = \boxed{8\sqrt{5}}$$

look for groups of 2

1 per group moves outside

Example 3 The same method works for other roots.

Find the cube root of 320 ($\sqrt[3]{320}$)

$$\begin{array}{r|l} 2 & 320 \\ 2 & 160 \\ 2 & 80 \\ 2 & 40 \\ 2 & 20 \\ 2 & 10 \\ & 5 \end{array}$$

$$\sqrt[3]{320} = \sqrt[3]{2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot5} = 2\cdot2\sqrt[3]{5} = \boxed{4\sqrt[3]{5}}$$

groups of 3

Try these on your own

1. $\sqrt{24}$
2. $\sqrt[3]{24}$
3. $\sqrt{512}$
4. $\sqrt[3]{512}$
5. $\sqrt[4]{512}$
6. $\sqrt{420}$
7. $\sqrt{450}$

Solutions

1. $2\sqrt{6}$
2. $2\sqrt[3]{3}$
3. $16\sqrt{2}$
4. 8
5. $4\sqrt[4]{2}$
6. $2\sqrt{105}$
7. $15\sqrt{2}$

Solving Quadratic Equations by factoring

procedure: 1 solve for zero (set to zero)
2 factor
3 set each factor to zero

Example 1 Solve $15x^2 + x = 2$

$15x^2 + x = 2$
$15x^2 + x - 2 = 0$
$(3x - 1)(5x + 2) = 0$

$3x - 1 = 0$ $\quad$ $5x + 2 = 0$
$3x = 1$ $\quad$ $5x = -2$
$\boxed{x = \frac{1}{3}}$ $\quad$ $\boxed{x = \frac{-2}{5}}$

Example 2 Solve $12x^2 - 45x - 12 = 0$

$12x^2 - 45x - 12 = 0$
$3(4x^2 - 15x - 4) = 0$
$3(4x + 1)(x - 4) = 0$

$3 \neq 0$ $\quad$ $4x + 1 = 0$ $\quad$ $x - 4 = 0$
$4x = -1$ $\quad$ $\boxed{x = 4}$
$\boxed{x = \frac{-1}{4}}$

Example 3 Solve $6x^3+50x^2+16x=0$

$6x^3+50x^2+16x=0$
$2x(3x^2+25x+8)=0$
$2x(x+8)(3x+1)=0$

$2x=0$ $\boxed{x=0}$

$x+8=0$ $\boxed{x=-8}$

$3x+1=0$
$3x=-1$
$\boxed{x=-\frac{1}{3}}$

Example 4 Solve $x(12x+1)=3$

the equation <u>must</u> be set to zero

$x(12x+1)=3$
$12x^2+x=3$
$12x^2+x-3=0$
$(4x+3)(3x-1)=0$

$4x+3=0$
$4x=-3$
$\boxed{x=-\frac{3}{4}}$

$3x-1=0$
$3x=1$
$\boxed{x=\frac{1}{3}}$

Solve by factoring practice problems

answers

1. $3(2x-5)(4x+3)=0$ — ① $\frac{5}{2}, -\frac{3}{4}$
2. $y^2-10y+24=0$ — ② $4, 6$
3. $\frac{c^2}{20}-\frac{c}{4}+\frac{1}{5}=0$ — ③ $1, 4$
4. $\frac{5x^2}{6}-\frac{7x}{2}+\frac{2}{3}=0$ — ④ $\frac{1}{5}, 4$
5. $n(2n-3)=2$ — ⑤ $-\frac{1}{2}, 2$
6. $x^2-6x=x(8+x)$ — ⑥ 0
7. $n(3+n)=n^2+4n$ — ⑦ 0
8. $12x^2+5x-2=0$ — ⑧ $\frac{1}{4}, -\frac{2}{3}$
9. $x(x-3)=x^2+5x+7$ — ⑨ $-\frac{7}{8}$
10. $25x^2-40x+16=0$ — ⑩ $\frac{4}{5}$

Solving Quadratic Equations
The square root method

If a quadratic equation cannot be factored, there are several methods that can be used to find the solution.

The square root method

The procedure:

1. isolate the $(\quad)^2$
2. take the square root of both sides
3. solve for the variable

<u>note</u>: don't forget the $\pm$

Example 1 Solve $(4x-1)^2=5$

$$(4x-1)^2=5$$

$$\sqrt{(4x-1)^2}=\sqrt{5}$$

$$4x-1=\pm\sqrt{5}$$

$$4x=1\pm\sqrt{5}$$

$$\boxed{x=\frac{1\pm\sqrt{5}}{4}=\frac{1}{4}\pm\frac{1}{4}\sqrt{5}}$$

Example 2

Solve $3(x-1)^2+7=12$

$$3(x-1)^2+7=12$$
$$-7 \quad -7$$
$$3(x-1)^2=5$$
$$(x-1)^2=\frac{5}{3}$$
$$x-1=\pm\sqrt{\frac{5}{3}}$$
$$\boxed{x=1\pm\sqrt{\frac{5}{3}}}$$

this form needs to adjusted.

$$1\pm\sqrt{\frac{5}{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$$

$$\boxed{1\pm\frac{\sqrt{15}}{3}} \text{ or } \boxed{\frac{3\pm\sqrt{15}}{3}}$$

try these on your own (see next page)

Square Root Method practice Problems

Add the proper constant to create a perfect square trinomial. Find the trinomials.

	answers	
① $y^2 + 2y +$ ____ $= (\quad)^2$	1	$(y+1)^2$
② $x^2 - 8x +$ ____ $= (\quad)^2$	16	$(x-4)^2$
③ $n^2 + 5n +$ ____ $= (\quad)^2$	$\frac{25}{4}$	$(n+\frac{5}{2})^2$
④ $y^2 - y +$ ____ $= (\quad)^2$	$\frac{1}{4}$	$(y-\frac{1}{2})^2$

Solve each problem using the square root method

⑤ $2(y-3)^2 = 8$	$1, 5$
⑥ $(y+4)^2 - 4 = 23$	$-4 \pm 3\sqrt{3}$
⑦ $(4x+9)^2 = 6$	$\frac{-9 \pm \sqrt{6}}{4}$
⑧ $(z-6)^2 = 18$	$6 \pm 3\sqrt{2}$

$\boxed{\text{Completing the Square}}$

procedure: format $ax^2+bx+c=0$

① $a=1$, so divide by a

② move the constant term to the right

③ add $\left(\frac{b}{2}\right)^2$ to both sides

④ The result is $(\quad)^2 = \#$

⑤ now follow the square root method to finish

goal: put the equation into square root method form.

$\boxed{\text{Example 1}}$ (easy one) $a=1$

$x^2+4x-7=0$

$x^2+4x=7$

$x^2+4x+\underline{4} = 7+\underline{4}$

$\left(\frac{4}{2}\right)^2=(2)^2=4$

$\underbrace{x^2+4x+4}=7+4$

$(x+2)^2=11$

$x+2=\pm\sqrt{11}$

$\boxed{x=-2\pm\sqrt{11}}$

note: the perfect square is always $\left(x+\frac{b}{2}\right)^2$ by design

Example 2 (medium)

$$2x^2 + 4x - 7 = 0$$

$$\frac{2x^2}{2} + \frac{4x}{2} - \frac{7}{2} = \frac{0}{2}$$

$$x^2 + 2x - \frac{7}{2} = 0$$

$$x^2 + 2x + \boxed{1} = \frac{7}{2} + \boxed{1}$$

$$\boxed{} = \left(\frac{2}{2}\right)^2 = 1^2 = 1$$

note: $\frac{b}{2} = 1$

$$(x+1)^2 = \frac{7}{2} + 1$$

$$(x+1)^2 = \frac{7}{2} + \frac{2}{2} = \frac{9}{2}$$

$$x + 1 = \pm\sqrt{\frac{9}{2}}$$

$$x = -1 \pm \frac{3}{\sqrt{2}}$$

$$x = -1 \pm \frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{-1 \pm \frac{3\sqrt{2}}{2}}$$

completing the square

Example 3 (Difficult)

$$3x^2-7x+11=0$$

$$\frac{3x^2}{2}-\frac{7x}{2}+\frac{11}{2}=\frac{0}{2}$$

$$x^2-\frac{7}{2}x+\frac{11}{2}=0$$

$$x^2-\frac{7}{2}x=-\frac{11}{2}$$

$$x^2-\frac{7}{2}x+\boxed{\frac{49}{4}}=-\frac{11}{2}+\boxed{\frac{49}{4}}$$

$$\boxed{}=\left(\frac{1}{2}\left(\frac{-7}{2}\right)\right)^2=\left(\frac{\frac{-7}{2}}{2}\right)^2=\left(\frac{-7}{4}\right)^2=\frac{49}{16}$$

neater format

$$x^2-\frac{7}{2}x+\frac{49}{4}=-\frac{11}{2}\cdot\frac{8}{8}+\frac{49}{16}$$

$$\left(x-\frac{7}{4}\right)^2=\frac{-88+49}{16}$$

$$x-\frac{7}{4}=\pm\sqrt{\frac{-39}{16}} \qquad x=\frac{7}{4}\pm\frac{\sqrt{39}\,i}{4}$$

$$x=\frac{7\pm i\sqrt{39}}{4}$$

Completing the square practice problems

1. $x^2 - 2x - 2 = 0$
2. $x^2 + 3x - 2 = 0$
3. $y^2 + y - 7 = 0$
4. $2x^2 + 14x - 1 = 0$
5. $6x^2 - 3 = 6x$
6. $3x^2 - 4x = 4$
7. $2y^2 + 12y + 3 = 0$
8. $x^2 - 7x - 1 = 0$
9. $4x^2 - 2x + 5 = 0$
10. $9x^2 - 36x = -40$

answers

1. $1 - \sqrt{3}, \; 1 + \sqrt{3}$
2. $\dfrac{-3 \pm \sqrt{17}}{2}$
3. $\dfrac{-1 \pm \sqrt{29}}{2}$
4. $\dfrac{-7 \pm \sqrt{51}}{2}$
5. $\dfrac{1 \pm \sqrt{3}}{2}$
6. $-\dfrac{2}{3}, \; 2$
7. $\dfrac{-6 \pm \sqrt{30}}{2}$
8. $\dfrac{7 \pm \sqrt{53}}{2}$
9. $\dfrac{1 \pm i\sqrt{19}}{4}$
10. $\dfrac{6 \pm 2i}{3}$

Using the Quadratic Formula to solve quadratic equations

traditional method $\quad ax^2+bx+c=0$

$$\boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

procedure
1. identify a, b, c
2. simplify (start with the radical)

Example 1 $\quad 4x^2-2x+3=0$

$a=4 \quad b=-2 \quad c=3$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2-4(4)(3)}}{2(4)}$$

$$x = \frac{2 \pm \sqrt{4-48}}{8}$$

$$x = \frac{2 \pm \sqrt{-44}}{8}$$

$$x = \frac{2 \pm 2\sqrt{11}\, i}{8} = \boxed{\frac{1 \pm \sqrt{11}\, i}{4}}$$

to simplify → all 3 must have a common factor

The Quadratic Formula
The Brazilian method

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

where D = discriminant
$D = b^2 - 4ac$

same as traditional but step by step

This method is used in multiple countries throughout the world.

process:
1. Find D (focus on signs)
2. find $\sqrt{D}$ (focus on $\sqrt{\ }$)
3. put it all together & simplify

Example 1 (Repeated) $\boxed{4x^2 - 2x + 3 = 0}$

$a = 4 \quad b = -2 \quad c = 3$

1. $D = b^2 - 4ac$
$D = (-2)^2 - 4(4)(3)$
$= 4 - 48$
$D = -44$

2. $\sqrt{D} = \sqrt{-44}$
$= 2i\sqrt{11}$

$$\begin{array}{r|l} 2 & 44 \\ \hline 2 & 22 \\ \hline & 11 \end{array}$$

3. $x = \dfrac{-b \pm \sqrt{D}}{2a}$

$$x = \frac{-(-2) \pm 2\sqrt{11}\,i}{2(4)}$$

$$x = \frac{2 \pm 2\sqrt{11}\,i}{8}$$

$$\boxed{x = \frac{1 \pm \sqrt{11}\,i}{4}}$$

Example 2

Solve
$$3x^2 - 7x + 11 = 0$$

$a = 3 \quad b = -7 \quad c = 11$

traditional quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(11)}}{2(2)}$$

$$x = \frac{7 \pm \sqrt{49 - 132}}{4}$$

$$x = \frac{7 \pm \sqrt{-83}}{4}$$

$$\boxed{x = \frac{7 \pm i\sqrt{83}}{4}}$$

Brazilian Method

$$D = b^2 - 4ac$$
$$D = (-7)^2 - 4(3)(11)$$
$$= 49 - 132 = -83$$

$$\sqrt{D} = \sqrt{-83} = i\sqrt{83}$$

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{+7 \pm i\sqrt{83}}{2(3)} = \boxed{\frac{7 \pm i\sqrt{83}}{6}}$$

Quadratic Formula practice problems

	answers
① $p^2+11p-12=0$	① $-12, 1$
② $5x^2-3=14x$	② $-\frac{1}{5}, 3$
③ $11n^2-9n=1$	③ $\frac{9\pm5\sqrt{5}}{22}$
④ $\frac{1}{8}x^2+x=\frac{5}{2}$	④ $-10, 2$
⑤ $(x+5)(x-1)=2$	⑤ $-2\pm\sqrt{11}$
⑥ $x(x+6)=2$	⑥ $-3\pm\sqrt{11}$
⑦ $10y^2+10y+3=0$	⑦ $\frac{-5\pm i\sqrt{5}}{10}$
⑧ $\frac{x^2}{2}-3=-\frac{9}{2}x$	⑧ $\frac{-9\pm\sqrt{105}}{2}$
⑨ $\frac{1}{8}x^2+x+\frac{5}{2}=0$	⑨ $-4\pm2i$
⑩ $x(7x+1)=2$	⑩ $\frac{-1\pm\sqrt{57}}{14}$

Hints: watch formatting $ax^2+bx+c=0$

multiply by the lcd to eliminate fractions before using the formula